Solved Problem on Magnetic Vector Potential:

Q.1 What current density would produce the vector potential $\vec{A} = K\hat{\phi}$ (where K is a constant), in cylindrical coordinates?

Soln: Ampere's law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{J} = \frac{1}{\mu_0}(\nabla \times \vec{B}) \quad — \quad (1)$$

$$\vec{B} = \nabla \times \vec{A} \quad — \quad (2)$$

Grad. in cylindrical coordinate is given by

$$\cancel{\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{}}$$

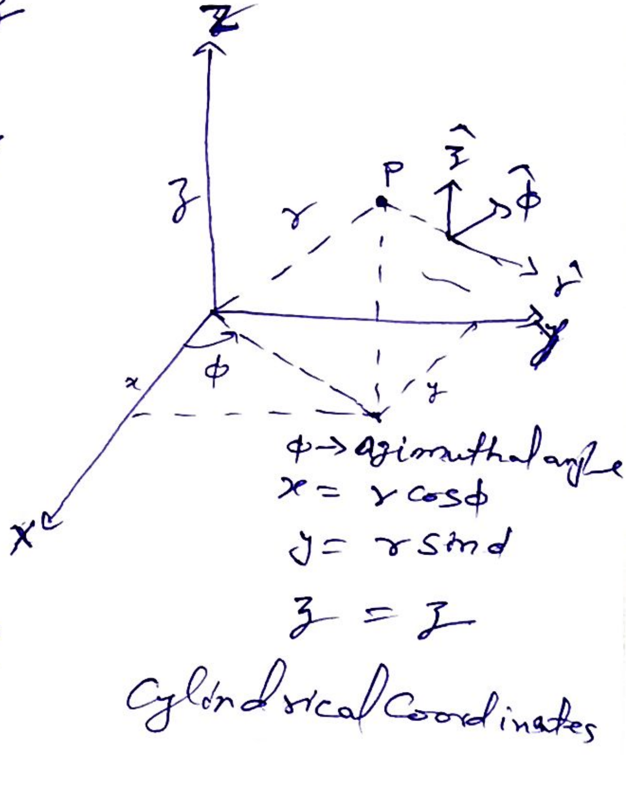
$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z} \quad — \quad (3)$$

vector Potential is given by

$$\vec{A} = K\hat{\phi} \quad — \quad (4)$$

~~Now from (2), (3) and (4)~~

$$\cancel{\vec{B} = \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi}}$$



$\phi \to$ azimuthal angle

$x = r\cos\phi$

$y = r\sin\phi$

$z = z$

Cylindrical Coordinates

$\vec{A}$ can be expressed in terms of cylindrical coordinates (r, ϕ, z) components A_r, A_ϕ, A_z as

$$\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z} \quad — \quad (5)$$

$\nabla\times\vec{A}$ in cylindrical coordinate canbe enpressed as (in general form)

$$\nabla\times\vec{A} = \frac{1}{r}\left[\hat{r}\left(\frac{\partial A_z}{\partial\phi} - \frac{\partial}{\partial z}(rA_\phi)\right) + \hat{\phi}\left(r\frac{\partial A_r}{\partial z} - r\frac{\partial A_z}{\partial r}\right) + \hat{z}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right)\right] \quad \text{—— (6)}$$

$\because$ $A_\phi = k$ { see eqn (4) and (5)

$A_r = 0 = A_z$

$$\nabla\times\vec{A} = \frac{1}{r}\left[\hat{z}\frac{\partial}{\partial r}(rk) - 0\right]$$

$$\text{or } \vec{B} = \nabla\times\vec{A} = \hat{z}\left(\frac{k}{r}\right) \quad \text{—— (7)}$$

in cylindrical coordinate

$\vec{B} = B_r\hat{r} + B_\phi\hat{\phi} + B_z\hat{z}$

$B_r = 0 = B_\phi$

$B_z = \frac{k}{r}$

from eqn (1) and (7)

$$\vec{J} = \frac{1}{\mu_0}(\nabla\times\vec{B}) \longrightarrow \text{see eq}^n \text{ (6)}$$

$$= \frac{1}{\mu_0}\frac{1}{r}\left[-\hat{\phi}\, r\,\frac{\partial(k/r)}{\partial r}\right]$$

$$= -\frac{\hat{\phi}}{\mu_0}k\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = \left(\frac{k}{\mu_0 r^2}\right)\hat{\phi}$$

$$\text{or } \boxed{\vec{J} = \left(\frac{k}{\mu_0 r^2}\right)\hat{\phi}}$$